Faculty of Engineering | **Philadelphia University** | Mechanical Eng. Dep.

Mechanical Vibration (620414)

First Exam

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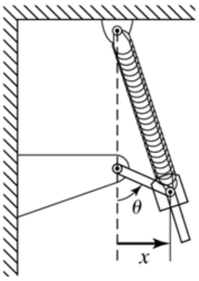
Student Name: **Student ID number:**

Problem #1: chose the correct answer and draw a table in the answer sheet and fill it with your choices. **(6marks)**

1. How many degree of freedom the system shown in the figure has

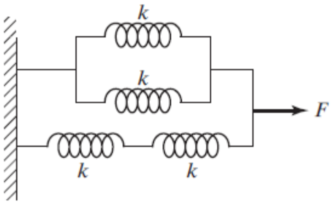


a. 0 b. 1 c.2 d.3 e. infinite

2. Fill the blank: the mass in vibrating system ________

a. Store kinetic energy
b. Store potential energy
c. Dissipate energy
d. a+b
e. Has no rule at all

3. Find the equivalent stiffness for the springs configuration shown in the figure where k = 100N/m



a. 100N/m b. 400N/m c.300N/m d.250N/m e. 40N/m

4. Find the natural frequency for single DoF system if k = 1000N/m and m=10kg.

a. 10 rad/s b. 10 Hz c.100 rad/s d.100 Hz e. 40N/m

5. Which of the following represents harmonic motion. Where a, b, c, and d are constants

a. $x(t) = ax + b$
b. $x(t) = a\ln(x^c) + b$
c. $x(t) = a\cos(bt) + c\sin(bt)$
d. $x(t) = a\cos(bt - d)$
e. c+d

6. if single DoF system with k = 1000N/m and m=10kg has the following response: $x(t) = A\cos(\omega_n t - \phi)$. If the initial displacement was 2m and the initial velocity equal zero. Then the values of A , Φ and ω_n will be respectively:

a. 10m, 30°, 10 rad/sec
b. 2m, 90°, 10 rad/sec
c. 2m, 0°, 10 rad/sec
d. 10m, 0°, 10 rad/sec
e. None of the above is correct

Q	1	2	3	4	5	6
Answer	B	A	D	A	E	C

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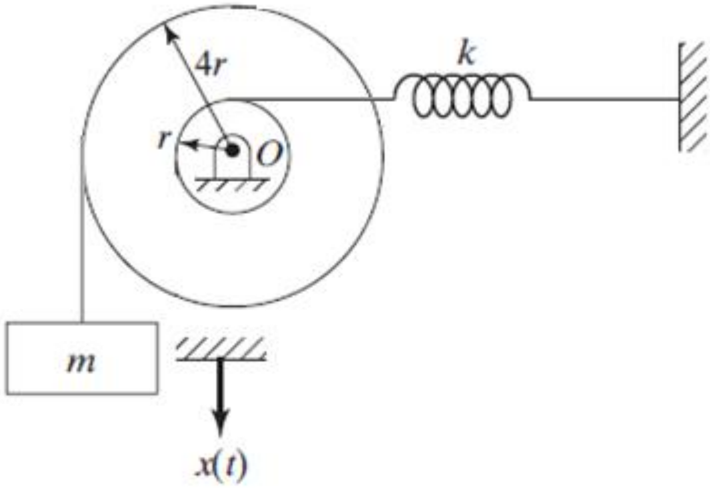
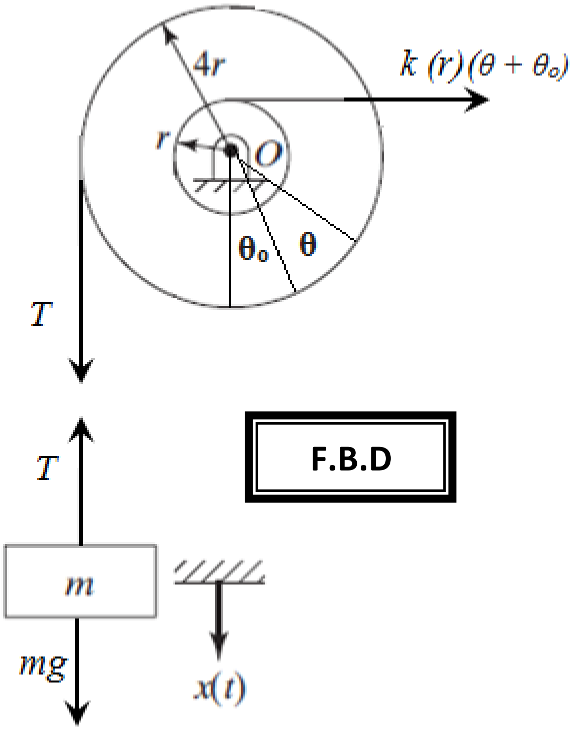
Problem #2: Draw the free-body diagram and derive the equation of motion using Newton s second law of motion for the systems shown in figure and find its natural frequency in terms of system parameters **(6 marks)**



Solution:

1. Equation of motion



a. when the mass is added to the system, a static deflection occurs in spring equivalent to the initial rotation. Let us name this initial rotation (θ_o).

b. the motion starts from the static point and let us name it (θ)

c. the force generated in spring is related to the angular deflection as follow:

$$F_s = k(r)(\theta + \theta_o)$$

d. the governing equation for the mass (m):

$$mg - T = m\ddot{x} \quad \text{.... Eq.1}$$

e. the governing equation for the pulley (J_o):

$$J_o\ddot{\theta} = T(4r) - k(r)(\theta + \theta_o)(r) \text{.... Eq.2}$$

f. according to static equilibrium(when θ=0): $mg(4r) = k(r\theta_o)(r) \Rightarrow \theta_o = \dfrac{4mg}{rk} \text{....} Eq.3$

g. substitute Eq.s 1 and 3 into Eq.2:

$$J_o\ddot{\theta} = \left(mg - m\ddot{x}\right)(4r) - kr^2\left(\theta + \frac{4mg}{rk}\right)$$

$$J_o\ddot{\theta} - mg(4r) + m\ddot{x}(4r) + kr^2\theta + (4r)mg = 0 \Rightarrow J_o\ddot{\theta} + m\ddot{x}(4r) + kr^2\theta = 0$$

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h. x to θ relation: $x = 4r\theta \rightarrow \ddot{x} = 4r\ddot{\theta}$ then the governing equation becomes:

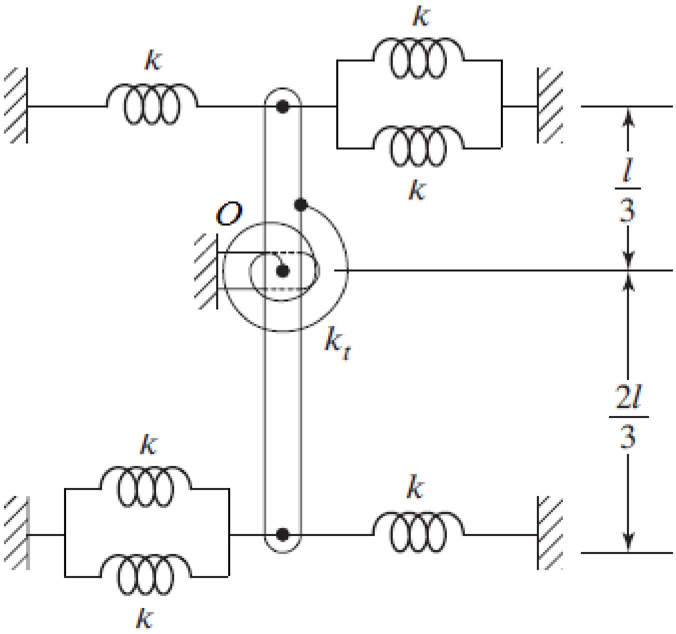
$$J_o\ddot{\theta} + m\ddot{\theta}(16r^2) + kr^2\theta = 0 \Rightarrow (J_o + 16mr^2)\ddot{\theta} + (kr^2)\theta = 0$$

$$\omega_n\sqrt{\frac{(kr^2)}{(J_o + 16mr^2)}}$$

Problem #3: if the system shown in the figure represents a rod connected to linear springs at its ends and a single torsional spring at point A. **(8 marks)**

If the rod has the following parameters:

1. m = 4.5 kg.
2. l = 1m
3. $J_O = \frac{1}{9}ml^2$



And the springs stiffness(s) are:

1. k = 150 N/m
2. k_t = 50 N.m/dgree

Find:

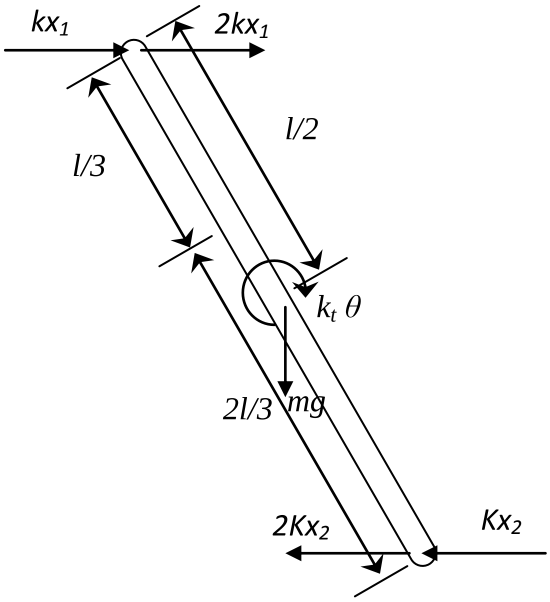
1. The mathematical model that govern this system
2. Find the natural frequency of this system
3. What is the response of this system if the initial displacement equal 10^o and the initial velocity equal zero.

Assume small angular deflection

Solution:

1. The coupled springs connected at the ends of the rod are connected as parallel springs. So, the equivalent stiffness for it will equal *k+k= 2k.*

2. The F.B.D for this system is shown as after giving it an initial excitation (θ)



3. Assume the positive direction is the counterclockwise which in this case, θ is positive and the other moment parts (springs and weight) are negative because they try to rotate the system clockwise.

4. Apply Newton's 2^{nd} law of motion to the system:

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$$\sum M_A = J_o \ddot{\theta}$$

$$J_o \ddot{\theta} = -2kx_1\left(\frac{l}{3}\right) - kx_1\left(\frac{l}{3}\right) - 2kx_2\left(\frac{2l}{3}\right) - kx_2\left(\frac{2l}{3}\right) - mg\left(\frac{l}{2} - \frac{l}{3}\right)\theta - k_t\theta$$

5. Rearrange the terms

$$J_o \ddot{\theta} + kx_1(l) + kx_2(2l) + mg\left(\frac{l}{6}\right)\theta + k_t\theta = 0$$

6. Now let us relate the translational distances (x_1 and x_2) to θ :

$$\sin(\theta) \approx \theta = \frac{x_1}{l/3} \Rightarrow x_1 = \theta\left(\frac{l}{3}\right)$$

$$\sin(\theta) \approx \theta = \frac{x_2}{2l/3} \Rightarrow x_2 = \theta\left(\frac{2l}{3}\right)$$

7. So the mathematical model becomes:

$$J_o \ddot{\theta} + k\left(\theta\frac{l}{3}\right)(l) + k\left(\theta\frac{2l}{3}\right)(2l) + mg\left(\frac{l}{6}\right)\theta + k_t\theta = 0$$

$$J_o \ddot{\theta} + \theta\left(k\frac{l^2}{3}\right) + \theta\left(k\frac{4l^2}{3}\right) + mg\left(\frac{l}{6}\right)\theta + k_t\theta = 0 \Rightarrow J_o \ddot{\theta} + \left\{k\frac{l^2}{3} + k\frac{4l^2}{3} + \frac{mg}{6} + k_t\right\}\theta = 0$$

8. Substitute the physical parameters of the system in the final format of the mathematical model:

$$J_O = \frac{1}{9}ml^2 = \frac{2}{9}kg.m^2 \Rightarrow \frac{1}{2}\ddot{\theta} + \left\{150\frac{1^2}{3} + 150\frac{4(1)^2}{3} + \frac{(4.5)(9.81)}{6} + 50\right\}\theta = 0 \Rightarrow 0.5\ddot{\theta} + 307.4\theta = 0$$

9. And hence, the natural frequency equal:

$$\omega_n = \sqrt{307.4/0.5} = 24.8\, rad/s$$

10. This system is a single DoF undamped free vibration system and the response of such system is given as:

$$\theta(t) = \theta_o \cos(\omega_n t) + \frac{\dot{\theta}_o}{\omega_n}\sin(\omega_n t) = A\cos(\omega_n t - \phi) = A_o \sin(\omega_n t + \phi)$$

1. $\phi = \tan^{-1}\left(\dfrac{\dot{\theta}_o}{\theta_o\omega_n}\right) = \tan^{-1}(0^o) = 0^o, \phi_o = \tan^{-1}\left(\dfrac{\theta_o\omega_n}{\dot{\theta}_o}\right) = \tan^{-1}(\infty) = \dfrac{\pi}{2}$

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2. $A = \sqrt{\theta_o^2 + \left(\frac{\dot{\theta}_o}{\omega_n}\right)^2} = \theta_o = 10^o = \frac{\pi}{18}$

So the solution can be:

1. $\theta(t) = \theta_o \cos(\omega_n t) + \frac{\dot{\theta}_o}{\omega_n}\sin(\omega_n t) = \frac{\pi}{18}\cos(24.8\,t)$
2. $\theta(t) = \frac{\pi}{18}\cos(24.8\,t)$
3. $\theta(t) = \frac{\pi}{18}\sin\left(24.8t + \frac{\pi}{2}\right)$